Solution

UNIT TEST -I (CHAPTER-1, 2, 3, 4, & 5)

Class 10 - Mathematics

Section A

1.

(c) 2

Explanation:

Since 5 + 3 = 8, the least prime factor of a + b has to be 2, unless a + b is a prime number greater than 2.
If a + b is a prime number greater than 2, then a + b must be an odd number. So, either a or b must be an even number. If a is even, then the least prime factor of a is 2, which is not 3 or 5. So, neither a nor b can be an even number. Hence, a + b cannot be a prime number greater than 2 if the least prime factor of a is 3 or 5.

2.

(b) coprime

Explanation:

We know that the co-prime numbers have no factor in common, or, their HCF is 1.

Thus, p^2 and q^2 have the same factor with exponent 2 each. which again will not have any common factor.

Thus we can conclude that p^2 and q^2 are co-prime numbers.

3. **(a)** ± 3

Explanation:

Let $\alpha,\ \beta$ are the zeroes of the given polynomial.
Given: $\alpha + \beta = \alpha\beta$
$\Rightarrow \frac{-b}{a} = \frac{c}{a}$
$\Rightarrow$ -b = -c
$\Rightarrow$ -(-27) = $3k^2$
$\Rightarrow$ k^2 = 9
$\Rightarrow$ k = ± 3

4.

(c) 5

Explanation:

p(y) = $5y^2$ + 13y + m.
Given one root of p(x) is reciprocal of other
i.e. If α = a then $\beta = \frac{1}{a}$
sum of roots $(\alpha + \beta) = \frac{-b}{a}$
a + $\frac{1}{a} = -\frac{13}{5}$
Product of roots $(\alpha \cdot \beta) = \frac{c}{a}$
$a \cdot \frac{1}{a} = \frac{m}{5}$.
1 = $\frac{m}{5}$.
m = 5

5.

(b) $\frac{5}{13}$

Explanation:

Let the fraction be $\frac{x}{y}$.
According to question
x + y = 18 ... (i)
And $\frac{x}{y+2} = \frac{1}{3}$

$\Rightarrow$ 3x = y + 2

$\Rightarrow$ 3x - y = 2 ... (ii)

On solving eq. (i) and eq. (ii), we get

x = 5, y = 13

Therefore, the fraction is $\frac{5}{13}$

6.

(b) $\frac{b^2}{4a}$

Explanation:

Since the roots are equal, we have D = 0

$\therefore b^2 - 4ac = 0 \Rightarrow 4ac = b^2 \Rightarrow c = \frac{b^2}{4a}$.

7. **(a)** 5 mins

Explanation:

Let the policeman catches the thief in n minutes.

Since the thief ran one minute before the policeman, therefore the time taken by thief before being caught

= (n + 1)minutes

Distance travelled by the thief in (n + 1) minutes

= 100 (n + 1) metres

In first minute, speed of policeman

= 100 m/minute.

In second minute, speed of policeman

= 110 m/minute

In third minute, speed of policeman

= 120 m/minute and so on.

$\therefore$ Speeds 100, 110, 120, ... form an A.P.

Total distance travelled by the policeman in n minutes = $\frac{n}{2}$[2 $\times$ 100 + (n - 1)10]

On catching the thief by policeman, distance travelled by the thief = Distance travelled by the policeman.

100(n + 1) = $\frac{n}{2}$[2 $\times$ 100 + (n - 1)10]

$\Rightarrow$ 100n + 100 = 100n + $\frac{n}{2}$(n - 1)10

$\Rightarrow$ 100 = n(n - 1)5 $\Rightarrow$ n^2 - n - 20 = 0

$\Rightarrow$ (n - 5)(n + 4) = 0 $\Rightarrow$ n - 5 = 0 or n + 4 = 0

$\Rightarrow$ n = 5 [as n = -4 is not possible]

$\therefore$ Time taken by the policeman to catch the thief

= 5 minutes

8. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$8x^2 + 3kx + 2 = 0$

Discriminant, D = b^2 - 4ac

D = $(3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$

For equal roots, D = 0

$9k^2 - 64 = 0$

$9k^2 = 64$

$k^2 = \frac{64}{9}$

k = $\pm\frac{8}{3}$

So, both A and R are true and R is the correct explanation of A.

9.

(d) A is false but R is true.

Explanation:

We have,

$a_n = a + (n - 1)d$

$a_{21} - a_7 = \{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$

a + 20d - a - 6d = 84

14d = 84

$d = \frac{18}{14} = 6$

d = 6

So, A is false but R is true.

Section B

10. Since, α and β are the zeroes of the quadratic polynomial

$f(x) = x^2 - p(x+1) - c = x^2 - px - (p+c)$

So A=1 B=-p,C=-(p+c)

Sum of the zeroes $\alpha + \beta = -\frac{B}{A} = P$

Product of the zeroes $\alpha\beta = \frac{C}{A} = -(p + c)$

(α + 1)(β + 1)

= αβ + α + β + 1

= αβ + (α + β) + 1

=-(p+c)+p+1

=-p-c-p+1

=1-c

Hence proved

OR

Compare $f(x) = 5x^2 - 7x + 1$ with $ax^2 + bx + c$ we get,

a = 5, b = - 7 and c= 1

Since α and β are the zeros of $5x^2 - 7x + 1$, we have

$\alpha+\beta=-\frac{(b)}{a}=-\frac{(-7)}{5}=\frac{7}{5}$

$\alpha\beta=\frac{c}{a}=\frac{1}{5}$

$\therefore \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$=\frac{\frac{7}{5}}{\frac{1}{5}}$

$=\frac{7}{5}\times\frac{5}{1}$

= 7

11. x - y = 8.................(1)

3 x - 3 y = 16.............(2)

Here, $a_1 = 1, b_1 = -1, c_1 = -8$

$a_2 = 3, b_2 = -3, c_2 = -16$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of linear equation is inconsistent.

Section C

12. $f(x) = (x+3)\left(2x^2 - 3x + a\right)$

$g(x) = (x-2)\left(3x^2 + 10x - b\right)$

since $(x+3)(x-2)$ is the HCF of $f(x)$ and $g(x)$

$\therefore x - 2$ is a factor of $2x^2 - 3x + a \ldots (i)$

and $x + 3$ is a factor of $3x^2 + 10x - b \ldots\ldots$(ii)

From (i) follow that 2 is a zero of $2x^2 - 3x + a$

$\Rightarrow 2 \times 2^2 - 3 \times 2 + a = 0$

$\Rightarrow 8 - 6 + a = 0$

$\Rightarrow 2 + a = 0 \Rightarrow a = -2$

From (ii), it follows that

-3 is a zero of $3x^2 + 10x - b$

$\Rightarrow 3x(-3)^2 + 10x(-3) - b = 0$

$\Rightarrow 27 - 30 - 6 = 0$

$\Rightarrow -3 - b = 0 \Rightarrow b = -3$

The values of a and b are -2 and -3 respectively.

13. Given, $x^2 + k(4x + k - 1) + 2 = 0$

$x^2 + 4kx + k^2 - k + 2 = 0$.

Here, a =1, b = 4k, c = k^2-k + 2 = 0

Now equation has real roots, D = 0

i,e. $b^2 - 4ac = 0$

$(4k)^2 - 4 \times 1 \times (k^2-k+2) = 0$

$16k^2 - 4k^2 + 4k - 8 = 0$

$12k^2 + 4k - 8 = 0$

$4(3k^2 + k - 2) = 0$

$3k^2 +3k - 2k - 2 = 0$

3k(k+1)-2(k+1) = 0

(3k-2)(k+1) = 0

k = $\frac{2}{3}$ or k = -1

14. First APs

63, 65, 67, ...

Here, a = 63

d = 65 - 63 = 2

$\therefore$ nth term = 63 + (n - 1)2 $\because a_n = a + (n - 1)d$

Second APs

3, 10, 17, ...

Here, a = 3

d = 10 - 3 = 7

$\therefore$ nth term = 3 + (n - 1)7 $\because a_n = a+(n - 1)d$

If the n th terms of two APs are equal then

63 + (n - 1)2 = 3 + (n - 1)7

$\Rightarrow$ (n - 1)2 - (n - 1)7 = 3 - 63

$\Rightarrow$ (n - 1) (2 - 7) = -60

$\Rightarrow$ (n - 1) (-5) = -60

$\Rightarrow n - 1 = \frac{-60}{-5}$

$\Rightarrow$ n - 1 = 12

$\Rightarrow$ n = 12 + 1

$\Rightarrow$ n = 13

Hence, for n = 13th terms of the two APs are equal

OR

According to question we observe that 56 is the first integer between 50 and 500 which is divisible by 7.

Also, when we divide 500 by 7 the remainder is 3. Therefore, 500 - 3 = 497 is the largest integer divisible by 7 and lying between 50 and 500. Thus, we have to find the number of terms in an A.P. with first term = 56,

last term = 497 and common difference = 7 (as the numbers are divisible by 7).

Let there be n terms in the A.P. Then,

$a_n = 497$

$\Rightarrow$ a + (n - 1) d = 497

$\Rightarrow \quad 56 + (n-1) \times 7 = 497$ ...[Because,a = 56 and d = 7]

$\Rightarrow$ 7n + 49 = 497

$\Rightarrow$ 7n = 448

$\Rightarrow$ n = 64

Threfore, there are 64 integers between 50 and 500 which are divisible by 7.

Section D

15. i. Point of intersection of graph of polynomial, gives the zeroes of the polynomial.
    $\therefore$ zeroes = -4 and 7

    ii. Since, zero's are α = -4, β = 7
    $\alpha + \beta$ = -4 + 7 = 3
    $\alpha\beta$ = -4 $\times$ 7 = -28
    P(x) = x^2 - (Sum of zeroes)x + product of zeroes
    P(x) = x^2 - 3x + (-28)
    P(x) = x^2 - 3x - 28

    iii. Product of zeroes = -4 $\times$ 7
    = -28

    OR

    a is a non-zero real number, b and c are any real numbers c.

16. i. (18 + x) (12 + x) = 2(18 $\times$ 12)

    ii. x^2 + 30x - 216 = 0

    iii. Solving : x^2 + 30x - 216 = 0
    $\Rightarrow$ (x + 36)(x - 6) = 0
    x $\neq$ -36 $\therefore \Rightarrow$ x = 6.
    new dimensions are 24 cm $\times$ 18 cm

    OR

    If (18 + x)(12 + x) = 220
    then x^2 + 30x - 4 = 0
    Here D = 900 + 16 = 916 which is not a perfect square.
    Thus we can't have any such rational value of x.

Section E

17. Let the cost price of the tea set and the lemon set be ₹ x and ₹ y respectively.

Loss on the tea set = ₹ $\frac{5x}{100}$ = ₹ $\frac{x}{20}$

Gain on the lemon set = ₹ $\frac{15y}{100}$ = ₹ $\frac{3y}{20}$

$\therefore$ Net gain = ₹ $\frac{3y}{20} - \frac{x}{20}$

$\frac{3y}{20} - \frac{x}{20} = 7$

$\Rightarrow 3y - x = 140$ ....(i)

Gain on the tea set = ₹ $\frac{5x}{100}$ = ₹ $\frac{x}{20}$

Loss on the lemon set = ₹ $\frac{10y}{100}$ = ₹ $\frac{y}{10}$

$\therefore$Net gain = ₹ $\left(\frac{x}{20} + \frac{y}{10}\right)$

$\frac{x}{20} + \frac{y}{10} = 13$

$\Rightarrow x + 2y = 260$ ......(ii)

By adding equations (i) and (ii), we get

$\Rightarrow 5y = 400$

$\Rightarrow y = 80$

Substituting $y = 80$ in (ii), we get $x = 100$.

$\therefore$ the actual price of the tea set is ₹ 100 and that of the lemon set is ₹ 80.

OR

Let the fixed charge be Rs. x and additional charge by Rs. y.

According to question,

x + (7 - 3)y = 27

or x + 4y = 27 ....(i)

and x + (5 - 3)y = 21

x + 2y = 21 ....(ii)

On substracting (i) and (ii), we get

2y = 6

y=3

putting y in (i),

x + 4(3) = 27

x = 15

∴ x = Rs. 15 and y = Rs. 3

∴ Fixed charge = Rs. 15

∴ Charge for each extra day = Rs. 3

18. Given that, a = 2, d = 8 and S_n = 90.

As, $S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[4 + (n-1)8]$

90 = n[2 + (n - 1)4]

90 = n[2 + 4n - 4]

90 = n(4n - 2) = $4n^2$ - 2n

$4n^2$ - 2n - 90 = 0

$4n^2$ - 20n + 18n - 90= 0

4n(n - 5) + 18(n - 5) = 0

(n - 5)(4n + 18) = 0

Either n = 5 or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, n = 5

a_n = a + (n - 1)d

a_5 = 2 + (5 - 1)8

= 2 + 4(8)

= 2 + 32 = 34

Solution

UNIT I (CHAPTER - 1, 2, 3, 4 &5) UNIT TEST 2

Class 10 - Mathematics

Section A

1.

(b) an irrational number

Explanation:

an irrational number

2.

(d) Irrational

Explanation:

$\sqrt{p}$ is an irrational number because the square root of every prime number is an irrational number. (for example $\sqrt{3}$ is an irrational number)

3.

(b) $\frac{b^2-2ac}{c^2}$

Explanation:

We have to find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Given α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c

$\alpha+\beta=\frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^3}$ = $\frac{-b}{a}$

$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$ = $\frac{c}{a}$

We have,

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2-\frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{\beta}{\alpha\beta}+\frac{\alpha}{\beta\alpha}\right)^2-\frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2-\frac{2}{\alpha\beta}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{\frac{-b}{a}}{\frac{c}{a}}\right)^2-\frac{2}{\frac{c}{a}}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{-b}{a}\times\frac{a}{c}\right)^2-\frac{2}{\frac{c}{a}}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{-b}{c}\right)^2-\frac{2a}{c}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{b^2}{c^2}\right)-\frac{2a\times c}{c\times c}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{b^2}{c^2}\right)-\frac{2ac}{c^2}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left(\frac{b^2-2ac}{c^2}\right)$

4.

(d) -5

Explanation:

x^2+5x+8

$\alpha+\beta=\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$

$=\frac{-5}{1}$

= -5

5.

(c) 2

Explanation:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

Taking,

$\frac{3}{6} = \frac{-1}{-\text{k}}$

$\Rightarrow \frac{1}{2} = \frac{1}{\text{k}}$

$\Rightarrow$ k = 2

$\frac{-1}{-\text{k}} = \frac{8}{16}$

$\Rightarrow \frac{1}{\text{k}} = \frac{1}{2}$

$\Rightarrow$ k = 2

So, the answer is k = 2

6.

(b) 36

Explanation:

Let the digit at units place be x and the digit at tens place e be y, then the number = 10y + x

Now, according to the question, $\frac{10y+x}{y+x} = \frac{4}{1}$

$\Rightarrow$10y + x = 4y + 4x

$\Rightarrow$6y = 3x$\Rightarrow$x = 2y ...(i)

Also, x = 3 + y$\Rightarrow$2y = 3 + y [From (i)]

$\Rightarrow$y = 3 and x = 6

$\therefore$Required number = 36

7. **(a)** -8

Explanation:

The given equation is of the form: $ax^2 + bx + c = 0$, where; a = 2, b = - 4 and c = 3.

Therefore, the discriminant (D) is given as $D = b^2 - 4ac$

$D = (-4)^2 - (4 \times 2 \times 3) = 16 - 24 = -8$

8.

(d) A is false but R is true.

Explanation:

We have, $x^2 + 3x + 1 = (x - 2)^2 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow 7x - 3 = 0$

it is not of the form $ax^2 + 6x + c = 0$

So, A is false but R is true.

9. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

10. $\beta + \alpha = -1$ and $\alpha\beta = -6$

$\therefore$ required sum of zeroes $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{1}{6}$

Required product of zeroes $= \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{-6}$

$\therefore$ required quadratic polynomial is

$k\left(x^2 - \frac{1}{6}x - \frac{1}{6}\right)$ or $\left(6x^2 - x - 1\right)$

OR

We have

f(x) = $5x^2 - 4 - 8x = 5x^2 - 8x - 4$

$= 5x^2 - 10x + 2x - 4$

= 5x(x - 2) + 2(x - 2)

f(x) = 0 ⇒ (x - 2) (5x + 2) = 0

⇒ x - 2 = 0 or 5x + 2 = 0

$\therefore x = 2, \frac{-2}{5}$

So, the zeroes of f(x) are 2 and $\frac{-2}{5}$

Verification:

Sum of zeroes $= 2 + \left(\frac{-2}{5}\right) = \frac{10-2}{5} = \frac{8}{5} = -\frac{\text{Coeff. of } x}{\text{coeff. of } x^2}$

Product of zeroes $= 2 \times \left(\frac{-2}{5}\right) = \frac{-4}{5} = \frac{\text{Constant term}}{\text{coeff. of } x^2}$

11. Condition for coincident lines,

$a_1/a_2 = b_1/b_2 = c_1/c_2$;

No, given pair of linear equations are

$\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Comparing with ax + by + c = 0;

Here, $a_1 = 1/2$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;

$a_1 / a_2 = 1/8$

$b_1 / b_2 = 1/8$

$c_1 / c_2 = 32/25$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines

Hence, the given pair of linear equations has no solution.

Section C

12. If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$ then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{6} = \frac{a}{b}$

$\Rightarrow 6 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 6b^2 = a^2$ ....(i)

⇒ 6 divides a^2 [∵ 6 divides $6b^2$]

⇒ 6 divides a

Let $a = 6c$ for some integer c

putting $a = 6c$ in (i), we get

$a^2 = 36c^2$

$6b^2 = 36c^2 \quad [6b^2 = a^2]$

$\Rightarrow b^2 = 6c^2$

⇒ 6 divides b^2 [∵ 6 divides $6c^2$]

⇒ 6 divides b [∵ 6 divides $b^2 = 6$ divides b]

Thus, 6 is a common factors of a and b

But, this contradicts the fact that a and b have no common factor other than 1

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence $\sqrt{6}$ is irrational.

13. $3x + 2y = 11$ ....(i)

$2x + 3y = 4$ ....(ii)

Multiplying (i) by 3 and (ii) by 2, we get

$9x + 6y = 33$ ....(iii)

$4x + 6y = 8$ ....(iv)

Subtracting (iv) from (iii), we ge

$5x = 25 \Rightarrow x = 5$

Substituing x = 5 equation in (i), we get

$3(5) + 2y = 11 \Rightarrow 15 + 2y = 11 \Rightarrow 2y = -4 \Rightarrow y = -2$

Hence, the solution of the given system of equations is x = 5, y = -2

14. Yes.

nth term of an A.P., $a_n = a + (n - 1)d$

$a_{30} = a + (30 - 1)d = a + 29d$

and $a_{20} = a + (20 - 1)d = a + 19d$ .....(i)

Now, $a_{30} - a_{20} = (a + 29d) - (a + 19d) = 10d$.

From given A.P. we have

Common difference, d = -7 - (-3) = -7 + 3 = -4

$a_{30} - a_{20} = 10(-4) = -40$

So, a = -3, d = -4

Therefore, nth term is, an = a + (n - 1)d = -3 + (n - 1)(-4)

= -3 - 4n + 4

= 1 - 4n

Therefore, $a_{20} = 1 - 4(20) = -79$

$a_{30} = 1 - 4(30) = -119$

OR

Let the first term be "a" and the common difference be "d" of the Arithmetic progression. According to the given information,

$a_4 = 9$

$\Rightarrow$ a+(4-1)d=9

$\Rightarrow$ a+3d=9...(1)

Also, $a_6+a_{13}=40$

$\Rightarrow$ a+(6-1)d+a+(13-1)d=40

$\Rightarrow$ a+5d+a+12d=40

$\Rightarrow$ 2a+17d=40...(2)

Multiplying equation (1) by 2,we get,

2a+6d=18..... (3)

Subtracting (3) from (2),

11d = 22

$\Longrightarrow$ d=$\frac{22}{11}$

$\Rightarrow$ d = 2

from (1),we have,

a+3d =9

$\Longrightarrow$ a = 9 - 3d (substitute value of d=2)

$\Longrightarrow$ a= 9-6=3

So, we get "a" and "d" as 3 ,2.

Therefore,the AP is a, a+d, a+2d, a+3d,..., that is 3,5,7,9,......

Section D

15. i. Graph of y = f(x) intersects X-axis at two distinct points. So we can say that no of zeros of y = f(x) is 2.
    ii. There will not be any zero if graph of f(x) does not intersect x- axis.
    iii. $x^2 + (a + 1) x + b$ is the quadratic polynomial.

    2 and -3 are the zeros of the quadratic polynomial.

    Thus, 2 + (-3) = $\frac{-(a+1)}{1}$

    $\Rightarrow$ $\frac{(a+1)}{1}$ = 1

    $\Rightarrow$ a + 1 = 1

    $\Rightarrow$ a = 0

    Also, 2 $\times$ (-3) = b

    $\Rightarrow$ b = -6

    OR

    If -4 is zero of given polynomial then,

$(-4)^2 - 2(-4) - (7p + 3) = 0$

$\Rightarrow 16 + 8 - 7p - 3 = 0$

$\Rightarrow 7p = 21$

$\Rightarrow p = 3$

16. i. Let the no of articles produced be x

Price of each article = 2x + 1

Price of all articles produced = ₹ 210

$x(2x + 1) = 210$

$2x^2 + x - 210 = 0$

ii. On solving

$2x^2 + x - 210 = 0$

$x = \frac{-1 \pm \sqrt{1-(4)(2)(-210)}}{2 \times 2}$

$x = \frac{-1 \pm \sqrt{1680}}{4}$

$x = \frac{-1 \pm \sqrt{1681}}{4}$

$x = \frac{-1 \pm 41}{4}$

$x = \frac{40}{4} = 10$

x = 10

$x = \frac{-42}{4}$

neglected as no of article's cannot be -ve

$\therefore$ no of articles = 10

$\therefore$ cost of each article = 2x + 1

$= 2 \times 10 + 1$

= ₹ 21

iii. Since cost of 1 article = 21

$\therefore$ cost of 15 article = 21×15

= ₹ 315

OR

Since 21 is the manufacturing cost of 1 article

$\therefore$ 1 is the manufacturing cost of $\frac{1}{21}$ article

$\therefore$ 1575 is the manufacturing cost $\frac{1}{21} \times 1575$

= 75 article

Section E

17. Let number of correct answers be x and

number of incorrect answers be y

$3x - y = 40$

$4x - 2y = 50$

Solving, we get x = 15, y = 5

$\therefore$ Total number of questions = 20

OR

Let the fixed charges of the car be ₹ x and the running charges be ₹ y/km

According to the given condition,

For a journey of 12 km, the charge paid is ₹ 89, we have

x + 12y = 89 ....(i)

And for a journey of 20 km, the charge paid is ₹145.

x + 20y = 145 ....(ii)

Subtracting equation (i) from equation (ii), we get

(x + 20y) - (x + 12y) = 145 -89

x + 20y - x - 12y = 56

20y - 12y = 56

$\Rightarrow 8y = 56$

$\Rightarrow y = \frac{56}{8} = 7$

Putting y = 7 in equation (i), we get

x + 12 × 7 = 89

⇒ x + 84 = 89

⇒ x = 89 - 84 = 5

∴ Total charges from travelling a distance of 30 km = x + 30y

= 5 + 30 × 7 = 5 + 210 = ₹ 215

Hence, total charges from travelling a distance of 30 km is ₹215.

18. Let a be the first term and d be the common difference of the given AP.

Now a_4 = a + (4 - 1)d

⇒ a_4 = a + 3d.

And, a_{17} = a + (17 - 1)d

⇒ a_{17} = a + 16d.

Then,

T_4 = a + 3d and T_{17} = a + 16d

Now, $\frac{T_4}{T_{17}} = \frac{1}{5}$

⇒ $\frac{a+3d}{a+16d} = \frac{1}{5}$

⇒ 5a + 15d = a + 16d

⇒ 5a - a + 15d - 16d = 0

⇒ 4a - d=0

⇒ 4a = d....(i)

Also, S_7 = 182

Where ,S_7 = $\frac{7}{2}[2a + (7-1)d]$

⇒ $\frac{7}{2}[2a + 6d] = 182$

⇒ $\frac{7\times2}{2}[a + 3d] = 182$

⇒ a + 3d = 26

⇒ a + 3(4a) = 26...[from (i)]

⇒ 13a = 26

⇒ a = 2

⇒ d = 4(2) = 8

Thus, We have

T_1 = 2

T_2 = T_1 + d = 2 + 8 = 10

T_3 = T_1 + 2d = 2 + 2(8) = 2 + 16 = 18

T_4 = T_1 + 3d = 2 + 3(8) = 2 + 24 = 26

Thus, the required AP is 2, 10, 18, 26,...

OR

Here, (-4) + (-1) + 2 + 5 + ... + x = 437.

Now,

-1 - (-4) = -1 + 4 = 3

2 - (-1) = 2 + 1 = 3

5 - 2 = 3

Thus, this forms an A.P. with a = -4, d = 3, l = x

S_n = $\frac{n}{2}$[2a + (n - 1)d]

⇒ 437 = $\frac{n}{2}$ [2 × (-4) + (n - 1) × 3]

⇒ 874 = n[-8 + 3n - 3]

⇒ 874 = n[3n - 11]

⇒ 874 = $3n^2$ - 11n

⇒ $3n^2$ - 11n - 874 = 0

⇒ $3n^2$ - 57n + 46n - 874 = 0

⇒ 3n(n - 19) + 46(n - 19) = 0

⇒ 3n + 46 = 0 or n = 19

⇒ $n = -\frac{46}{3}$ or n = 19

Numbers of terms cannot be negative or fraction.

⇒ n = 19

Now, $S_n = \frac{n}{2}[a + l]$

⇒ 437 = $\frac{19}{2}$ [-4 + x]

⇒ -4 + x = $\frac{437 \times 2}{19}$

⇒ -4 + x = 46

⇒ x = 50

Solution

UNIT TEST - II (CHAPTER - 6, 7, 8 & 9)

Class 10 - Mathematics

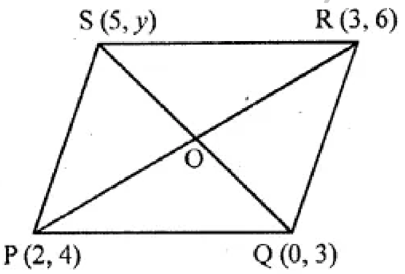
Section A

1. **(a)** similar but may not be congruent
   Explanation:
   Two Circles are said to be **congruent** if they have the **same radius**.. If radius is not given then they are congruent onle when if their radius are equal and all circles are **similar** to each other, regardless of their size or radius.
   so correct option is (c) similar but may not be congruent

2.
   (c) DC $\times$ AC
   Explanation:
   Since, triangles ABC and DCB are similar
   $\therefore \frac{AB}{AC} = \frac{DC}{DB} \Rightarrow$ AB $\times$ DB = DC $\times$ AC

3.
   (c) a
   Explanation:
   Distance between (a cos 25^o, 0) and (0, a cos65^o)
   $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
   $= \sqrt{(0 - a\cos 25^\circ)^2 + (a\cos 65^\circ - 0)^2}$
   $= \sqrt{a^2\cos^2 25^\circ + a^2\cos^2 65^\circ}$
   $= \sqrt{a^2[\cos^2 25^\circ + \cos^2 65^\circ]}$
   $= a\sqrt{\cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ}$
   $= a\sqrt{\sin^2 65^\circ + \cos^2 65^\circ}$
   $= a(\sqrt{1}) = a$

4.
   (b) 7
   Explanation:
   P(2, 4), Q(0, 3), R(3, 6) and S(5, y) are the vertices of parallelogram PQRS



   Join PR and QS which intersect at O
   $\therefore$ O is mid-point of PR and QS
   When O is mid point of PR then coordinates
   of O will be = $\left(\frac{2+3}{2}, \frac{4+6}{2}\right)$
   $= \left(\frac{5}{2}, 5\right)$
   O is mid-point of QS
   $\therefore 5 = \frac{y+3}{2} \Rightarrow y + 3 = 10$
   $\Rightarrow$ y = 10 - 3 = 7

5.

(d) $\sqrt{3}$

Explanation:

Given: $\sin\theta = \frac{\sqrt{3}}{2}$ and $\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\Rightarrow \cot^2\theta = \frac{4}{3} - 1$ [Given]

$\Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$

$\therefore \operatorname{cosec}\theta + \cot\theta = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

6.

(c) 45^o

Explanation:

At A = 45^o

$\sin A = \sin 45^o = \frac{1}{\sqrt{2}}$

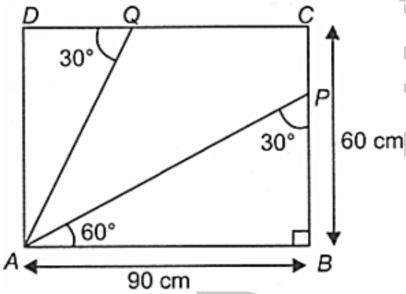
$\cos A = \cos 45^o = \frac{1}{\sqrt{2}}$

7. **(a)** 300 cm

Explanation:

In $\triangle$ADQ, $\frac{AD}{AQ} = \sin 30^o \Rightarrow \frac{AD}{AQ} = \frac{1}{2}$

$\Rightarrow$ AQ = 2AD $\Rightarrow$ AQ = $2 \times 60 = 120$ cm



In $\triangle$ABP, $\frac{AB}{AP} = \cos 60^o \Rightarrow \frac{AB}{AP} = \frac{1}{2}$

$\Rightarrow$ AP = 2AB $\Rightarrow$ AP = $2 \times 90 = 180$ cm

$\therefore$ AP + AQ = 300 cm

8.

(d) A is false but R is true.

Explanation:

$\sin\theta$ and $\operatorname{cosec}\theta$ are reciprocal of each other so $\sin\theta \times \operatorname{cosec}\theta = 1$

$\sin\theta \times \operatorname{cosec}\theta \neq \cot\theta$

9. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

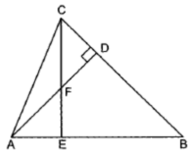
10. In $\triangle$ABC and $\triangle$AMP

$\angle$ABC = $\angle$AMP ...(i) [Each equal to 90^o]

∠BAC=∠MAP ...(ii) [Common angle]
△ABC ∼ △AMP (AA similarity criterion)

OR

Given Altitude AD and CE of △ABC intersects each other at the point F.
To Prove: $\triangle FDC \sim \triangle BEC$

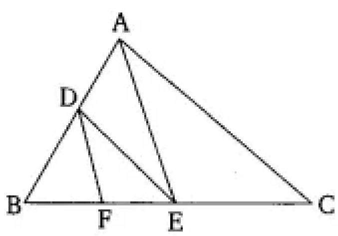


Proof: In △'s FDC and BEC, we have
∠FDC = ∠BEC = 90^0 [$\because AD \perp BC$ and $CE \perp AB$]
∠FCD = ∠ECB [Common angle]
Thus, by AA-criterion of similarity, we obtain $\triangle FDC \sim \triangle BEC$.

11. 

The points of trisection means that the points which divide the line into three equal parts. From the figure, it is clear that C, and D are these two points. Let C (x_1, y_1) and D (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., BC = CD = DA
Let BC = CD = DA = k, Point C divides BC and CA as: BC = kCA = CD + DA = k + k = 2k
Hence the ratio between BC and CA is: $\frac{BC}{CA} = \frac{k}{2k} = \frac{1}{2}$
Therefore, point C divides BA internally in the ratio 1:2 then by section formula we have that if a point P(x, y) divides two points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n then, the point (x, y) is given by $(\mathrm{x}, \mathrm{y}) = \left(\frac{\mathrm{mx_2+nx_1}}{\mathrm{m+n}}, \frac{\mathrm{my_2+ny_1}}{\mathrm{m+n}}\right)$
Therefore C(x, y) divides B(–2, –3) and A(4,–1) in the ratio 1:2, then
$C(x,y) = \left(\frac{(1\times4)+(2\times-2)}{1+2}, \frac{(1\times-1)+(2\times-3)}{1+2}\right)$
$C(x,y) = \left(\frac{4-4}{1+2}, \frac{-1-6}{1+2}\right)$
$C(x,y) = \left(0, \frac{-7}{3}\right)$
Point D divides the BD and DA as:DA = kBD = BC + CD = k + k = 2k
Hence the ratio between BD and DA is: $\frac{BD}{DA} = \frac{2k}{k} = \frac{2}{1}$
The point D divides the line BA in the ratio 2:1
So now applying section formula again we get,
$D(x,y) = \left(\frac{(2\times4)+(1\times-2)}{2+1}, \frac{(2\times-1)+(1\times-3)}{2+1}\right)$
$D(x,y) = \left(\frac{8-2}{3}, \frac{-2-3}{3}\right)$
$D(x,y) = \left(\frac{6}{3}, \frac{-5}{3}\right)$
$D(x,y) = \left(2, \frac{-5}{3}\right)$

Section C

12. 

Given: DE || AC and DF || AE
To Prove: $\frac{BF}{FE} = \frac{BE}{EC}$
Proof: In ΔABC, DE||AC
$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ...(i) [By Basic proportionality Theorem]
In ΔBAE, DF||AE
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ...(ii) [[By Basic proportionality Theorem]
From (i) and (ii) we get

$\frac{BE}{EC} = \frac{BF}{FE}$

Hence proved.

13. Given: P(2, 2) is equidistant from the points A(-2, k) and B(-2k, -3),

We have, AP = BP

$AP^2 = BP^2$

$(2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 + (2 + 3)^2$

$16 + 4 + k^2 - 4k = 4 + 4k^2 + 8k + 25$

$20 + k^2 - 4k = 29 + 4k^2 + 8k$

$3k^2 + 12k + 9 = 0$

$k^2 + 4k + 3 = 0$

$k^2 + 3k + k + 3 = 0$

k(k + 3) + 1(k + 3) = 0

(k + 1) (k + 3) = 0

k = -1, -3

For k = -1, we have,

AP = $\sqrt{(2+2)^2 + (2-k)^2} = \sqrt{16 + (2+1)^2} = \sqrt{25} = 5$

For k = -3, we have,

AP = $\sqrt{(2+2)^2 + (2+3)^2} = \sqrt{16 + 25} = \sqrt{41}$

OR

Given: A(3, -1), B(5, -1) and C(3, -3)

$AB = \sqrt{(5-3)^2 + (-1+1)^2} = \sqrt{2^2 + 0^2} = 2$

$BC = \sqrt{(5-3)^2 + (-1+3)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$AC = \sqrt{(3-3)^2 + (-1+3)^2} = \sqrt{2^2} = 2$

Clearly, AB = AC

Therefore, $\triangle$ABC is an isosceles triangle.

Now, $AB^2 = 4$, $BC^2 = 8$ and $AC^2 = 4$

Therefore, $BC^2 = AB^2 + AC^2$

Therefore, $\triangle$ABC is right angled also.

Hence, $\triangle$ABC is right isosceles triangle.

14. We have to find the value of:

$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$

Now,

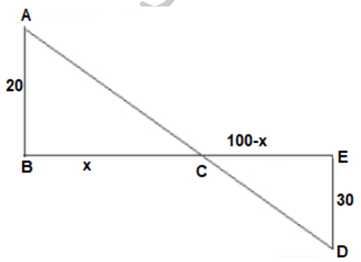
$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 4\left[\frac{1}{16} + \frac{1}{4}\right] - 3\left[\frac{1}{2} - 1\right] - \frac{3}{4}$

$= 4\left(\frac{1+4}{16}\right) - 3\left(\frac{-1}{2}\right) - \frac{3}{4} = \frac{5}{4} + \frac{3}{2} - \frac{3}{4}$

$= \frac{5+6-3}{4} = \frac{8}{4} = 2$

Section D

15. i. 

$\triangle$ABC $\sim$ $\triangle$DEC

$\frac{20}{30} = \frac{x}{100-x}$

2000 - 20x = 30x

2000 = 50x

x = 40 m

ii. AA

iii. 60 metres

OR

AD = AC + CD

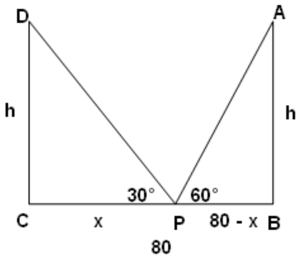
$= \sqrt{20^2 + 40^2} + \sqrt{60^2 + 30}^2$

$= \sqrt{400 + 1600} + \sqrt{3600 + 900}$

$= \sqrt{2000} + \sqrt{4500}$

$\Rightarrow 20\sqrt{5} + 30\sqrt{5}$

$\Rightarrow 50\sqrt{5}$ m

16. 

Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x) . Also, ∠APB = 60^o and ∠DPC =30^o

In right angled triangle DCP,

tan 30^o = $\frac{CD}{CP}$

⇒ $\frac{h}{x} = \frac{1}{\sqrt{3}}$

⇒ $h = \frac{x}{\sqrt{3}}$ ....(1)

In right angled triangle ABP,

Tan 60^o = AB/AP $\frac{AB}{AP}$

⇒ $\frac{h}{80-x} = \sqrt{3}$

⇒ $h = \sqrt{3}(80 - x)$

⇒ $\frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

⇒ x = 3(80 – x)

⇒ x = 240 – 3x

⇒ x + 3x = 240

⇒ 4x = 240

⇒ x = 60

Height of the tower, h = x/√3 = 60/√3 = 20√3.

Thus, the position of the point P is 60 m from C and the height of each tower is 20√3 m.

Section E

17. Let Q(x, 0) be a point on x–axis which lies on the perpendicular bisector of AB.

Therefore, QA = QB

⇒ $QA^2 = QB^2$

⇒ $(-5 - x)^2 + (-2 - 0)^2 = (4 - x)^2 + (-2 - 0)^2$

⇒ $(x + 5)^2 + (-2)^2 = (4 - x)^2 + (-2)^2$

⇒ $x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$

⇒ 10x + 8x = 16 – 25

⇒ 18x = –9

⇒ x = $\frac{-9}{18} = \frac{-1}{2}$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now, $QA^2 = \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2$

$= \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$

⇒ $QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

⇒ QA = $\sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

Now, $QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$

$\Rightarrow QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

and $AB = \sqrt{(4+5)^2 + [-2-(-2)]^2} = \sqrt{(9)^2}$ = 9 units

⇒ AB = 9 units

As QA = QB

So, Δ QAB is an isosceles Δ.

18. Given,

cos A - sin A = m

$\Rightarrow$ (cos A - sin A)^2 = m^2

$\Rightarrow$ cos^2 A + sin^2 A - 2 cos A sin A = m^2

$\Rightarrow$ 1 -2 cos A sin A = m^2 ...(i)

Also given,

cos A + sin A = n

$\Rightarrow$ (cos A + sin A)^2 = n^2

$\Rightarrow$ cos^2 A + sin^2 A + 2 cos A sin A= n^2

$\Rightarrow$ 1 + 2 cos A sin A = n^2 ...(ii)

Adding (i) & (ii), we get :-

(1 - 2cosAsinA) + (1 + 2cosAsinA) = m^2 + n^2

$\Rightarrow$ m^2 + n^2 = 2 .....(iii)

Similarly, on subtracting equation (ii) from (i) we get :-

- 4 cos A sin A = m^2 - n^2 ...(iv)

Now, L.H.S.

$= \frac{m^2-n^2}{m^2+n^2}$

$= \frac{-4\cos A\sin A}{2}$ [from (iii) & (iv)]

= - 2 sin A cos A

So, $\frac{m^2-n^2}{m^2+n^2} = -2SinACosA$ ......(v)

Now,

-2sinAcosA

$= \frac{-2\sin A\cos A}{1}$

$= \frac{-2\sin A\cdot\cos A}{\sin^2 A+\cos^2 A}$ ($\because$ sin^2 A + cos^2 A = 1)

$= \frac{\frac{-2\sin A\cos A}{\sin A\cos A}}{\frac{\sin^2 A+\cos^2 A}{\sin A\cdot\cos A}}$

$= \frac{-2}{\frac{\sin^2 A}{\sin A\cdot\cos A}+\frac{\cos^2 A}{\sin A\cdot\cos A}}$

$= \frac{-2}{\tan A+\cot A}$

So, -2sinAcosA $= \frac{-2}{\tan A+\cot A}$ ......(vi)

Now, from (v) & (vi) ,

$\frac{m^2-n^2}{m^2+n^2}$ = - 2 sin A . cos A $= \frac{-2}{\tan A+\cot A}$ Hence, Proved.

OR

We have,

$\text{LHS} = \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$\Rightarrow$ LHS $= \frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$

$\Rightarrow$ LHS $= \frac{\tan A}{\frac{\tan A-1}{\tan A}} + \frac{1}{\tan A(1-\tan A)}$

$\Rightarrow$ LHS $= \frac{\tan^2 A}{\tan A-1} + \frac{1}{\tan A(1-\tan A)}$

$\Rightarrow$ LHS $= \frac{\tan^2 A}{\tan A-1} - \frac{1}{\tan A(\tan A-1)}$

$\Rightarrow$ LHS $= \frac{\tan^3 A-1}{\tan A(\tan A-1)}$ [Taking LCM]

$\Rightarrow$ LHS $= \frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)}$ [$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$\Rightarrow$ LHS $= \frac{\tan^2 A+\tan A+1}{\tan A}$

$\Rightarrow$ LHS $= \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A} + \frac{1}{\tan A}$

$\Rightarrow$ LHS = tanA + 1 + cotA [since (1/tanA) =cotA].

= (1 + tanA + cotA)

$\therefore$ $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA ....(1)

Now, 1 + tanA + cotA

= 1 + $\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

= 1 + $\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}$ = 1 + $\frac{1}{\sin A\cos A}$ [$\because \text{Sin}^2 A + \text{Cos}^2 A = 1$]

= 1 + cosecAsecA

So, 1 + tanA + cotA = 1+ cosecAsecA.......(2)

From (1) and (2), we obtain

$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA = 1 + cosecAsecA

Solution

UNIT II (CHAPTER - 6, 7, 8 & 9) TEST- 2

Class 10 - Mathematics

Section A

1. **(a)** 35 cm

Explanation:

$\triangle ABC \sim \triangle DEF$

$\frac{ar(\triangle\ ABC)}{ar(\triangle\ DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$

$\frac{ar(\triangle\ ABC)}{ar(\triangle\ DEF)} = \left(\frac{BC}{EF}\right)^2$

$\frac{9}{25} = \left(\frac{21}{EF}\right)^2$

$\frac{9}{25} = \frac{441}{EF^2}$

$EF^2 = \frac{441 \times 25}{9}$

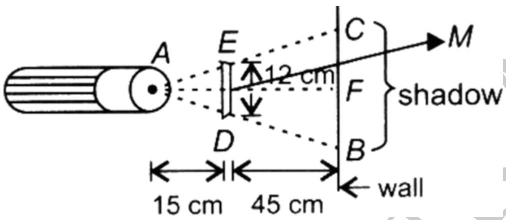
$EF = \sqrt{\frac{441 \times 25}{9}}$

$EF = \frac{21 \times 5}{9}$

EF = 35 cm.

2. **(c)** 48 cm

Explanation:



In $\triangle$ADM and $\triangle$ABF

$\angle$DAM = $\angle$BAF (Common)

$\angle$DMA = $\angle$BFA (Corresponding angles)

$\therefore$ $\triangle$ADM ~ $\triangle$ABF (By AA similarity)

$\Rightarrow \frac{AD}{AB} = \frac{DM}{BF} = \frac{AM}{AF}$ or $\frac{DM}{BF} = \frac{AM}{AF}$

$\Rightarrow \frac{6}{BF} = \frac{15}{60} \Rightarrow$ BF = 24 cm

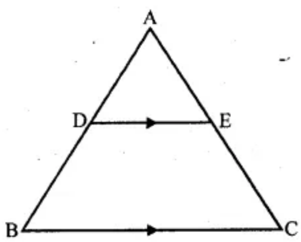
$\therefore$ BC = BF + FC = 2BF = 48 cm ($\because$ CF = BF)

3. **(d)** 4.4 cm

Explanation:

In ΔABC, DE || BC

AD : DB = 3 : 1, EA = 3.3 cm



Let EC = x

$\because$ In $\triangle$ABC, DE || BC

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{1} = \frac{3.3}{x}$

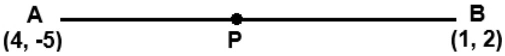
$\Rightarrow x = \frac{3.3}{3} = 1.1\text{cm}$

$\therefore$ AC = AE + EC = 3.3 + 1.1 = 4.4 cm

4.

(c) $\left(\frac{13}{7}, 0\right)$

Explanation:



Coordinate of P

$P\left(\frac{5\times1+2\times4}{5+2}, \frac{5\times2+2(-5)}{5+2},\right.$

$P\left(\frac{5+8}{7}, \frac{10-10}{7}\right)$

$P\left(\frac{13}{7}, 0\right)$

5.

(c) 9

Explanation:

Given, $\cot\theta = \frac{4}{3}$

$\therefore \frac{(5\sin\theta+3\cos\theta)}{(5\sin\theta-3\cos\theta)} = \frac{5+3\cot\theta}{5-3\cot\theta}$ [dividing num. and denom. by sin θ]

= $\frac{\left(5+3\times\frac{4}{3}\right)}{\left(5-3\times\frac{4}{3}\right)} = \frac{(5+4)}{(5-4)} = \frac{9}{1} = 9$

6.

(c) 30^o

Explanation:

We know that

Tan A = $\frac{CB}{AB}$

Tan A = $\frac{15}{15\sqrt{3}}$

Tan A = $\frac{1}{\sqrt{3}}$

Tan A = Tan 30^o

on comparing the t-ratios

A = 30^o

7. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\because$ cosec$^2\theta$ - cot$^2\theta$ = 1

$\Rightarrow$ (cosecθ - cotθ) (cosecθ + cotθ) = 1

$\Rightarrow$ (cosecθ - cotθ) = $\frac{1}{\operatorname{cosec}\theta+\cot\theta}$

$\therefore$ It is clear that cosecθ - cotθ and cosecθ + cotθ are reciprocal of each other.

8.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

9.

(c) $\frac{4}{5}$

Explanation:

We know that the sum of all the angles on one side of a straight line is 180°.

These angles are said to be in linear pairs.

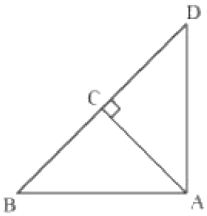
Therefore, using the figure, we get

$\theta + \phi + 90^\circ = 180^\circ$

Therefore, $\theta = 90^\circ - \phi$ ...(a)
Using trigonometric ratio in $\triangle ABC$, we get
$\sin\theta = \frac{4}{5}$ ...(b)
Using equation (a) in equation (b), we get
$\sin(90^\circ - \phi) = \frac{4}{5}$
We know that for any angle theta,
$\sin(90^\circ - \theta) = \cos\theta$.
Therefore, we get
$\cos\phi = \frac{4}{5}$
Therefore, the correct option is option is $\frac{4}{5}$

Section B

10. Given: $\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD.
To Prove: AB2 =BC $\times$ BD



Proof: In $\triangle$ADB and $\triangle$CAB
$\angle DAB = \angle ACB = 90^\circ$
$\angle ABD = \angle CBA$ [common angle]
$\angle ADB = \angle CAB$ [remaining angle]
So, $\triangle$ADB $\sim$ $\triangle$CAB (by AAA similarity)
Therefore $\frac{AB}{CB} = \frac{BD}{AB}$
$\Rightarrow$ $AB^2 = CB \times BD$

OR

According to question it is given that ABCD is a rectangle and p is the midpoint of DC.
$\therefore$ AD = BC = 9 cm
QC = BQ + BC = 7 + 9 = 16 cm
PC = $\frac{1}{2}$CD $\Rightarrow$ PC = 12 cm
In right $\triangle$PCQ using Pythagoras theorem
$PQ^2 = QC^2 + PC^2$
$PQ^2 = 16^2 + 12^2 = 400 \Rightarrow PQ = 20$ cm
In right $\triangle$ABQ using Pythagoras theorem
$AQ^2 = AB^2 + BQ^2 \Rightarrow AQ^2 = 24^2 + 7^2 = 625$
$\Rightarrow$ AQ = 25 cm
In right $\triangle$ADP using Pythagoras theorem
$AP^2 = AD^2 + DP^2 \Rightarrow AP^2 = 9^2 + 12^2$
$\Rightarrow AP^2 = 81 + 144$
$\Rightarrow AP^2 = 255$
AP = 15 cm
In $\triangle$APQ,
$AP^2 = 15^2 = 225$
$PQ^2 = 20^2 = 400 \Rightarrow AP^2 + PQ^2 = 625$
Also, $AQ^2 = 25^2 = 625 \Rightarrow AQ^2 = AP^2 + PQ^2$
$\therefore$ $\triangle$APQ is a right angled $\triangle$(using converse of BPT)
$\therefore$ $\angle$APQ = 90°

11. Since (2, 1) and (1, -2) are equidistant from (x, y), therefore,
Distance of (2, 1) from (x, y) = Distance of (1, -2) from (x, y)

$\sqrt{(x-2)^2+(y-1)^2}=\sqrt{(x-1)^2+(y+2)^2}$

$(x - 2)^2 + (y - 1)^2 = (x - 1)^2 + (y + 2)^2$

$x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 4 + 4y$

4 - 4x + 1 - 2y = 1 - 2x + 4 + 4y

2x + 6y = 0

x + 3y = 0

Section C

12. i. Resorting to the given figure we observe that In Δ's AEC and ADB,

$\angle$AEC = $\angle$ADB = 90° [$\because CE \perp AB$ and $BD \perp AC$]

and, $\angle$EAC = $\angle$DAB [Each equal to $\angle$ A]

Therefore, by AA-criterion of similarity, we obtain

ΔAEC~ ΔADB

ii. We have,

ΔAEC ~ ΔADB [As proved above]

$\Rightarrow \quad \frac{CA}{BA}=\frac{EC}{DB}$ {For similar triangles corresponding sides are proportional}

$\Rightarrow \quad \frac{CA}{AB}=\frac{CE}{DB}$

13. Let the coordinates of third vertex be (x,y)

Each length of equilateral triangle = $\sqrt{(0-0)^2+(3+3)^2}=\sqrt{6^2}=6$

Since the triangle is equilateral, therefore length of each side = 6.

Thus, Distance between (x, y) and (0, 3) = 6

$\sqrt{(x-0)^2+(y-3)^2}=6$

$(x - 0)^2 + (y - 3)^2 = 36$

$x^2 + (y - 3)^2 = 36$ .... (i)

Also, Distance between (x, y) and (0, -3) = 6

$\sqrt{(x-0)^2+(y+3)^2}=6$

$x^2 + (y + 3)^2 = 36$ .... (ii)

From (i) and (ii), we get,

$x^2 + (y - 3)^2 = x^2 + (y + 3)^2$

$\Rightarrow$ 12y = 0

$\Rightarrow$ y = 0

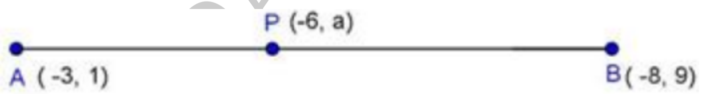
From (i), $x^2 + (0 - 3)^2 = 36$

$x^2 = 36 - 9 = 27$

x = $\pm 3\sqrt{3}$

Thus, the third vertex is $(\pm 3\sqrt{3}, 0)$

OR



Let P(-6, a) divides the join of A(-3, 1) and B(-8, 9) in the ratio k:1

Then, the coordinates of P are

$\left(\frac{-8k-3}{k+1}, \frac{9k+1}{k+1}\right)$

But, $\frac{-8k-3}{k+1}=-6$

$\Rightarrow$ -8k - 3 = -6k - 6

$\Rightarrow$ -8k + 6k = -6 + 3

$\Rightarrow$ -2k = -3

$\Rightarrow \quad k=\frac{3}{2}$

Hence, P divides AB in the ratio 3:2

Again,

$\frac{9k+1}{k+1}=a$

Substituting $k=\frac{3}{2}$

We get,

$\frac{9\times\frac{3}{2}+1}{\frac{3}{2}+1}=a$

$\Rightarrow \frac{\frac{29}{2}}{\frac{5}{2}}=a$

$\Rightarrow \quad \frac{29}{5}=a$

$\therefore \quad a=\frac{29}{5}$

14. We have, sin (A+B) =1

$\Rightarrow$ sin (A+B)$=\sin 90^\circ$

$\Rightarrow$ A+B = 90°...(i)

and, cos (A - B)$=\cos 30^\circ$

$\Rightarrow \quad A-B=30^\circ$ ...(ii)

Adding (i) and (ii), we get

(A+B) + (A-B)

$=90^\circ+30^\circ \Rightarrow 2A=120^\circ \Rightarrow A=60^\circ$

Putting $A=60^\circ$ in (i), we get

$60^\circ+B=90^\circ$

$\Rightarrow B=30^\circ$

Hence, A = 60^o and B = 30^o

Section D

15. i. cos θ = $\frac{60}{120}$

cos θ = $\frac{1}{2}$

θ = 60^o

ii. tan 60^o = $\sqrt{3}$

iii. tan θ = $\frac{AB}{BC}$

tan 60^o = $\frac{AB}{60}$

$\sqrt{3}$ = $\frac{AB}{60}$

AB = 60 $\times$ 1.732

AB = 103.9 m

OR

$\angle$A = 90 - θ

$\angle$A = 90 - 60

$\angle$A = 30^o

tan 30^o = $\frac{1}{\sqrt{3}}$

16. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

Thus, distance d = 600 $\times$ $\frac{1}{60}$ = 10 miles

ii. Now, tan 20^o = $\frac{BB'}{B'A}$ = $\frac{h}{10+x}$ ....eq(1)

And tan 60^o = $\frac{CC'}{C'A}=\frac{BB'}{C'A}$ = $\frac{h}{x}$

x = $\frac{h}{tan60^o}$ = $\frac{h}{\sqrt{3}}$

Putting the value of x in eq(1), we get,

tan 20^o = $\frac{h}{10+\frac{h}{\sqrt{3}}}=\frac{\sqrt{3}h}{10\sqrt{3}+h}$

0.364(10$\sqrt{3}$ + h) = $\sqrt{3}$ h

6.3 + 0.364 h = 1.732 h

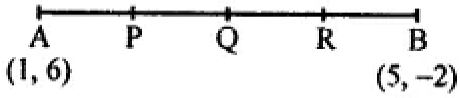
1.368 h = 6.3

h = 4.6

Thus, the altitude 'h' of the airplane is 4.6 miles.

Section E

17. Points P, Q and R in order divide a line segment joining the points A(1, 6) and B(5, -2) in 4 equal parts.



P divides AB in the ratio of 1:3 Let coordinates of P be (x, y), then

$x = \frac{mx_2+nx_1}{m+n} = \frac{1\times5+3\times1}{1+3}$

$= \frac{5+3}{4} = \frac{8}{4} = 2$

$y = \frac{my_2+ny_1}{m+n} = \frac{1\times(-2)+3\times6}{1+3}$

$= \frac{-2+18}{4} = \frac{16}{4} = 4$

∴ Coordinates of P are (2, 4)

Similarly,

Q divides AB in 2:2 or 1:1 and Q is midpoint of AB.

∴ Coordinates of Q will be $\left(\frac{1+5}{2}, \frac{6-2}{2}\right)$

or $\left(\frac{6}{2}, \frac{4}{2}\right)$ or (3, 2)

and R divides AB in the ratio of 3:1

Coordinates of R will be

$\left(\frac{3\times5+1\times1}{3+1}, \frac{3\times(-2)+1\times6}{3+1}\right)$

or $\left(\frac{15+1}{4}, \frac{-6+6}{4}\right)$ or $\left(\frac{16}{4}, \frac{0}{4}\right)$ or $(4, 0)$

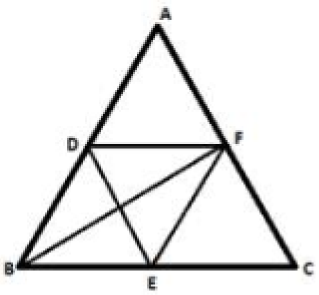
OR

Let the vertices of the triangle are A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)

D,E AND F are the mid points of sides AB, BC AND AC

Given, D(1,2),E(0,-1)and F(2,-1).

Draw DE, DF, FE and BF



As D and F are mid points of AB and AC

∴ DF || BE

E and F are mid points of BC and AC

∴ EF || BD

Hence, DBEF is a parallelogram

We know that, the diagonals of a parallelogram bisect each other.

That means, both diagonals have same mid - point.

Midpoint BF = Midpoint of DE

$\Rightarrow \left(\frac{x_2-2}{2}, \frac{y_2+1}{2}\right) = \left(\frac{1+0}{2}, \frac{2-1}{2}\right)$

On comparing both sides, we get

$\frac{x_2-2}{2} = \frac{1}{2}$ and $\frac{y_2+1}{2} = \frac{1}{2}$

⇒ x_2 - 2 = 1, y_2 + 1 = 1

∴ x_2 = 3, y_2 = 0

D is the midpoint of AB

D = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(1, 2) = $\left(\frac{x_1+3}{2}, \frac{y_1+0}{2}\right)$

⇒ x_1 = -1 and y_1 = 4

Now, F is the midpoint of AC

F = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

(2, -1) = $\left(\frac{-1+x_3}{2}, \frac{4+y_3}{2}\right)$

⇒ x_3 = 5 and y_3 = -6

The vertices of the triangle are (1, 2), (3, 0) and (5, - 6)

18. LHS $= \frac{\cot^2 A(\sec A-1)}{1+\sin A}$

$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$

$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$

$= \frac{\frac{\cos A\times\cos A}{1-\cos^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$ $[\because \sin^2 A+\cos^2 A=1]$

$= \frac{\frac{\cos A}{(1)^2-\cos^2 A}(1-\cos A)}{1+\sin A}$

$= \frac{\frac{\cos A}{(1+\cos A)(1-\cos A)}(1-\cos A)}{(1+\sin A)}$

$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$

RHS $= \sec^2 A\left[\frac{1-\sin A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\frac{1}{\cos A}}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\frac{\cos A+1}{\cos A}}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$

$= \frac{1}{\cos A\times\cos A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$

$= \frac{1-\sin A}{\cos A(1+\cos A)}$

Multiplying numerator and denominator by (1 + sinA)

$= \frac{(1-\sin A)}{\cos A(1+\cos A)}\times\frac{(1+\sin A)}{1+\sin A}$

$= \frac{(1)^2-\sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos A\times\cos A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$

∴ LHS = RHS

OR

We have, $asin\theta + bcos\theta = c$

On squaring both sides, we get

$(asin\theta + bcos\theta)^2 = c^2$

$(a \sin \theta)^2 + (b \cos \theta)^2 + 2(a \sin \theta)(b \cos \theta) = c^2$

⇒ $a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = c^2$

⇒ $a^2(1 - \cos^2 \theta) + b^2(1 - \sin^2 \theta) + 2ab \sin \theta \cos \theta = c^2$ $[\because sin^2\theta + cos^2\theta = 1]$

⇒ $a^2 - a^2 \cos^2 \theta + b^2 - b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = c^2$

⇒ $-a^2 \cos^2 \theta - b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = c^2 - a^2 - b^2$

Taking Negative common,

⇒ $a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta = a^2 + b^2 - c^2$

⇒ $(a \cos \theta)^2 + (b \sin \theta)^2 - 2(a \cos \theta)(b \sin \theta) = a^2 + b^2 - c^2$

⇒ $(acos\theta - bsin\theta)^2 = a^2 + b^2 - c^2$

⇒$acos\theta - bsin\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Hence proved, $acos\theta - bsin\theta = \sqrt{a^2 + b^2 - c^2}$

Solution

UNIT TEST - III (CHAPTER - 10, 11, 12, 13 & 14)

Class 10 - Mathematics

Section A

1.

(c) 40882.8 m^2

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{123^\circ}{360^\circ} \times \frac{22}{7} \times 138^2$

= 20441.4 m^2

Area covered by the man of the walking track in a day = 20441.4 + 20441.4

= 40882.8 m^2

2. **(a)** 5 : 1

Explanation:

Ratio of the total surface area to the lateral surface area = $\frac{\text{Total surface area}}{\text{Lateral surfice area}}$

= $\frac{2\pi r(h+r)}{2\pi rh}$

= $\frac{h+r}{h}$

= $\frac{(20+80)}{20}$

= $\frac{100}{20}$

= $\frac{5}{1}$

= 5 : 1

Hence, the required ratio is 5:1

3. **(a)** $\frac{x}{2\sqrt{\pi}}$

Explanation:

Let V_1 be the volume of the cylinder with radius r and height h, then

$V_1 = \pi r^2 h$ .... (i)

Now, let V_2 be the volume of the box, then

$V_2 = x^2 h$

It is given that $V_1 = 1/4\ V_2$. Therefore,

$\pi r^2 h = \frac{1}{4} x^2 h$

$\Rightarrow r^2 = \frac{x^2}{4\pi} \Rightarrow r = \frac{x}{2\sqrt{\pi}}$

4. **(a)** $\frac{\sum f_i x_i}{\sum f_i}$

Explanation:

The mean of discrete frequency distribution $\frac{x_i}{f_i}$; i = 1, 2, 3, ... n, will be $\frac{\sum f_i \times x_i}{\sum f_i}$

5.

(d) 26

Explanation:

mode = 3 median - 2 mean

= 3(30) - 2(32)

= 90 - 64

= 26

6. **(a)** $\frac{1}{2}$

Explanation:

$\frac{1}{2}$

7.

(d) 50^{o}

Explanation:

$\angle$ABC = 90^{o} [Angle in semicircle]

In $\triangle$ABC, we have

$\angle ACB + \angle CAB + \angle ABC = 180^{\circ}$

$\Rightarrow 50^{\circ} + \angle CAB + 90^{\circ} = 180^{\circ}$

$\Rightarrow \angle CAB = 40^{\circ}$

Now, $\angle CAT = 90^{\circ} \Rightarrow \angle CAB + \angle BAT = 90^{\circ}$

$\Rightarrow 40^{\circ} + \angle BAT = 90^{\circ} \Rightarrow \angle BAT = 50^{\circ}$

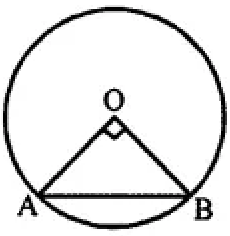
8.

(d) $10\sqrt{2}$

Explanation:

A chord subtends a right angle at its centre

Radius of the circle = 10 cm



$\therefore$ Chord AB $= \sqrt{r^2 + r^2}$

$= \sqrt{10^2 + 10^2}$

$= \sqrt{100 + 100} = \sqrt{200}$

$= \sqrt{100 \times 2} = 10\sqrt{2}$cm

9.

(d) A is false but R is true.

Explanation:

Area of a segment = Area of sector – Area of triangle

i.e., = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

Section B

10. Area of shaded region

= $\left[\pi(42)^2 - \pi(21)^2\right] \frac{300^{\circ}}{360^{\circ}}$

= $\frac{22}{7} \times 63 \times 21 \times \frac{5}{6}$.

= 3465 cm^2

11.

Classes	**Frequency**	**x_i**	**$f_i x_i$**
0-20	6	10	60
20-40	8	30	240
40-60	6	50	300
60-80	**f**	70	70**f**
80-100	5	90	450

mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$

50 = $\frac{1050 + 70f}{25 + f}$

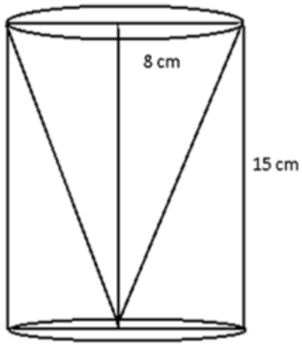
1250 + 50 f = 1050 + 70 f

1250 - 1050 = 70 f - 50 f

200 = 20 f

f = 10

Section C

12. 

Given that,

Height of the conical part (h) =Height of the cylindrical part (h) = 15 cm

Diameter of the cylindrical part = 16 cm

$\Rightarrow$ Radius of the cylindrical part (r) = 8 cm

Now,

Total Surface area of the remaining solid = CSA of the cylindrical part + CSA of the conical part + Area of cylindrical base.

$= 2\pi rh + \pi r\sqrt{r^2 + h^2} + \pi r^2$

$= \left[2 \times \frac{22}{7} \times 8 \times 15\right] + \left[\frac{22}{7} \times 8 \times \sqrt{8^2 + 15^2}\right] + \left[\frac{22}{7} \times (8)^2\right]$

= 754.286 + 427.4 + 201.1

= 1382.786 cm^2

13. Number of 50-p coins = 100.

Number of Rs. 1 coins = 70.

Number of Rs. 2 coins = 50.

Number of Rs. 5 coins = 30.

Therefore, the total number of outcomes = 100+70+50+30=250

i. Suppose E1 be the event of getting a Rs. 1 coin.

The number of favorable outcomes = 70.

Therefore, P(getting a Rs. 1 coin) = $P(E_1) = \frac{Number \text{ of outcomes favorable to } E_1}{\text{Number of all possible outcomes}} = \frac{70}{250}$

Thus, the probability that the coin will be a Rs. 1 coin is $\frac{7}{25}$.

ii. Suppose E_2 be the event of not getting a Rs. 5.

Number of favorable outcomes = 250 - 30 = 220

Therefore, P(not getting a Rs. 5 coin)

$= P(E_2) = \frac{\text{Number of outcomes favorable to } E_2}{\text{Number of all possible outcomes}} = \frac{220}{250} = \frac{22}{25}$

Thus, probability that the coin will not be a Rs. 5 coin is $\frac{22}{25}$.

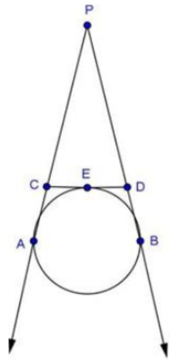
iii. Suppose E_3 be the event of getting a 50-p or a Rs. 2 coins.

Number of favorable outcomes = 100 + 50 = 150

Therefore, P(getting a 50-p or a Rs. 2 coin) = P(E_3) = $\frac{\text{Number of outcomes favorable to} E_3}{\text{Number of all possible outcomes}} = \frac{150}{250} = \frac{3}{5}$

Thus, probability that the coin will be a 50-p or a Rs. 2 coin is $\frac{3}{5}$.

14. We have,



AC = CE, BD = DE

And, AP = BP = 14 cm

$\therefore$ Perimeter of ΔPCD = PC + CD+ PD

$\Rightarrow$ Perimeter of ΔPCD = PC + (CE + ED) + PD

= (PC + CE) + (ED + PD)

= (PC + AC) + (BD + PD)

= PA + PB

= 14 + 14

= 28

$\therefore$ Perimeter of $\triangle$PCD = 28 cm.

Section D

15. i. Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525 \text{cm}^3$

ii. Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30} \text{cm}^3$

$\therefore$ Volume of four conical depressions

$= 4 \times \frac{11}{30} \text{cm}^3 = \frac{22}{15} \text{cm}^3 = 1.47 \text{cm}^3$

$\therefore$ Volume of the wood in the entire stand

$= 525 - 1.47 = 523.53 \text{cm}^3$

16. i. $\angle$NSA is the angle between tangent and radius at Point of contact and this is equal to 90^o.

ii. Distance between Sarika and Anju

NS = $\sqrt{13^2 - 5^2}$

= $\sqrt{169 - 25}$

= $\sqrt{144}$

NS = 12 m

iii. If $\angle$SNR = θ

then $\angle$RAS = 180 - θ

But, $\angle$NAS + $\angle$RAN = 180 - θ

2$\angle$NAS = 180 - θ {$\because$ $\angle$NAS = $\angle$RAN}

$\angle$NAS = 90 - $\frac{\theta}{2}$

OR

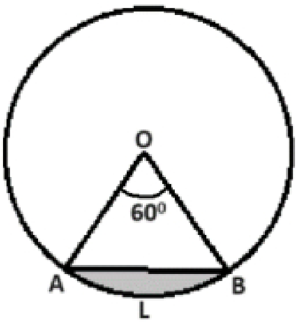
$\because$ NRAS is a quadrilateral

$\angle$N + $\angle$R + $\angle$A + $\angle$S = 360^o

$\angle$N + 90^o + $\angle$A + 90^o = 360^o

$\therefore$ $\angle$SNR + $\angle$SAR = 180^o

Section E

17. 

i. Length of the arc AB $= 2 \times \frac{22}{7} \times 21 \times \frac{60}{360}$

= 22 cm

ii. Area of sector OALB $= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360} = 231 \text{ cm}^2$

Area of $\triangle \text{OAB} = \frac{\sqrt{3}}{4} \times 21 \times 21 = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Area of minor segment $= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

or (231 - 190.95) = 40.05 cm^2

18.

Marks	**x**	**f**	$u = \frac{x-52\cdot5}{5}$	**fu**	**cf**
40 - 45	42.5	8	-2	-16	8
45 - 50	47.5	9	-1	-9	17
50 - 55	52.5	10	0	0	27
55 - 60	57.5	9	1	9	36
60 - 65	62.5	5	2	10	41
65 - 70	67.5	4	3	12	45
		45		6	

Mean = 52.5 + 5 × $\frac{6}{45}$ = 53.2 (approx)

Median = 50 + $\frac{5}{10}$ (22.5 - 17)

= 52.75

Solution

UNIT III (CHAPTER - 10, 11, 12, 13 & 14) TEST 2

Class 10 - Mathematics

Section A

1. **(a)** 13 cm

   Explanation:

   Radius of wheel = $\frac{91}{2}$ cm

   Angle between two adjoining spokes, $\theta = \frac{360^\circ}{22}$

   $\therefore$ Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

   = $\frac{360^\circ}{360^\circ \times 22} \times 2 \times \frac{22}{7} \times \frac{91}{2}$ = 13 cm

2. **(b)** $\frac{132}{7}$ cm^2

   Explanation:

   Angle of the sector is 60°

   Area of sector = $(\frac{\theta}{360^o}) \times \pi$r^2

   $\therefore$ Area of the sector with angle 60^o = $(\frac{60^o}{360^o}) \times \pi$r^2 cm^2

   = $(\frac{36}{6})\pi$ cm^2

   = 6 $\times$ $(\frac{22}{7})$ cm^2

   = $\frac{132}{7}$ cm^2

3. **(d)** $\frac{2h}{3}$

   Explanation:

   Height of cone = h

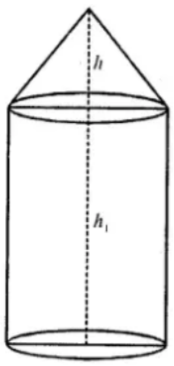
   Volume of solid = 3 $\times$ volume of cone

   Let h be the height of the cylinder and r be its radius, then

   Volume of cylinder and r be its radius, then

   Volume of cylinder = $\pi r^2 h_1$

   and volume of cone = $\left(\frac{1}{3}\right)\pi r^2 h_1$

   Then volume of solid = $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h$

   $= \pi r^2 \left(h_1 + \frac{1}{3}h\right)$

   Now $\pi r^2 \left(h_1 + \frac{1}{3}h\right) = 3 \times \frac{1}{3}\pi r^2 h = \pi r^2 h$

   $\Rightarrow h_1 + \frac{1}{3}h = h$ (comparing)

   $h_1 = h - \frac{1}{3}h = \frac{2}{3}h$

   Hence, height of cylinder = $\frac{2h}{3}$

4. **(a)** 142296

Explanation:

Space filled in the cube = $\left(\frac{7}{8} \times 22 \times 22 \times 22\right)$ cm^3 = (7 × 1331) cm^3

Radius of each marble = $\frac{0.5}{2}$ cm = $\frac{5}{20}$ cm = $\frac{1}{4}$ cm

Volume of each marble = $\frac{4}{3}\pi r^3$ = $\left(\frac{4}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$ cm3 = $\left(\frac{11}{24 \times 7}\right)$ cm^3

Number of marbles = $\left(\frac{7 \times 1331 \times 24 \times 7}{11}\right)$ = 142296

5.

(b) 24

Explanation:

Mean = 28

Mode = 16

Mode = 3 Median - 2 Mean

Hence, Median $= \frac{\text{Mode } + 2\text{Mean}}{3}$

$= \frac{16+2(28)}{3}$

$= \frac{16+56}{3}$

$= \frac{72}{3}$

= 24

6.

(d) 20

Explanation:

Class	Frequency	Cumulative frequency
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2} = \frac{67}{2} = 33.5$ which lies in the interval 125-145.

Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145.

Hence, the lower limit of modal class is 125.

∴ Required difference = upper limit of median class - lower limit of modal class

= 145 - 125 = 20

7.

(c) 0

Explanation:

Elementary events are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

∴ Number of Total outcomes = 36

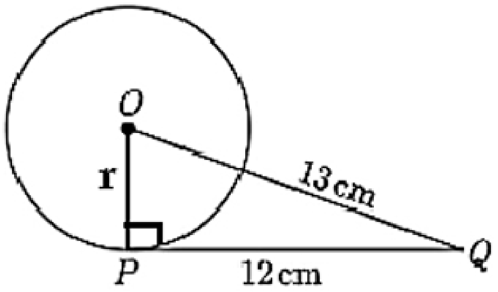
And Number of possible outcomes (sum of numbers appearing on die is 13) = 0

$\therefore$ Required Probability = $\frac{0}{36} = 0$

8.

(b) 5

Explanation:



$\because$ We know that

PQ $\perp$ OP

$\therefore$ $\triangle$QPO is right angled $\triangle$

$\therefore$ By python theory.

$QO^2 = QP^2 + OP^2$

$(13)^2 = (12)^2 + OP^2$

$r^2 = 169 - 144.$

$r^2 = 25$

r = 5 cm

9.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Area of the sector = $\frac{\theta}{360} \times \pi r^2$

= $\frac{60}{360} \times \frac{22}{7} \times 6 \times 6$

= $\frac{132}{7} = 18\frac{6}{7}$ cm^2

Section B

10. Side of equilateral triangle = 20 m

Length of rope = 14 m

$\theta = 60^o$

Area grazed by horse = $\frac{\pi r^2 \theta}{360^\circ}$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = \frac{308}{3}$ m^2 or 10267 m^2

Area of $\triangle$ = $\frac{\sqrt{3} \times 400}{4} = 100\sqrt{3}$ or 173 m^2

Required area $= (100\sqrt{3} - 10267)\text{m}^2$ or $70 \cdot 33$ m^2

11.

Number of days	Number of students (f_i)	Class mark (x_i)	$f_i x_i$
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39

Total	$\sum f_i = 40$		$\sum f_i x_i = 499$

Using the direct method,

$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{499}{40}$ = 12.475

Hence, the mean number of days a student was absent is 12.48

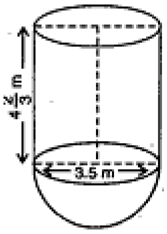
Section C

12. The diameter of the vessel = 3.5 m

$\therefore$ base radius of the toy = 1.75 m

The height of the cylindrical portion = $4\frac{2}{3}m = \frac{14}{3}m$

The volume of the vessel=volume of the cylindrical portion + volume of the hemispherical portion



=$\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left[h + \frac{2}{3}r\right]$

=$\frac{22}{7} \times (1.75)^2\left(\frac{14}{3} + \frac{2}{3} \times 1.75\right) m^3$

=$9.625 \times \frac{17.5}{3} m^3 = 56.15\, m^3$

Internal surface area of the solid = curved surface area of the hemispherical portion.

=$2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

=$2 \times \frac{22}{7} \times 1.75\left(\frac{14}{3} + 1.75\right) m^2$

=$\frac{11 \times 19.25}{3} m^2 = 70.58\, m^2$

13. When we throw a die, possible outcomes are 1,2,3,4,5 and 6.

i. Let E_1 be the event of getting a 3.

Then, the number of favourable outcomes = 1.

The favourable outcome is 3.

$\therefore$ P(getting a 3) = P(E_1) $= \frac{1}{6}$.

ii. Let E_2 be the event of getting a 5

Then, the number of favourable outcomes = 1.

The favourable outcome is 5.

$\therefore$ P(getting a 5) = P(E_2) $= \frac{1}{6}$.

iii. Let E_3 be the event of getting an odd number.

Number of favourable outcomes = 3.

The favourable outcomes are 1,3,5.

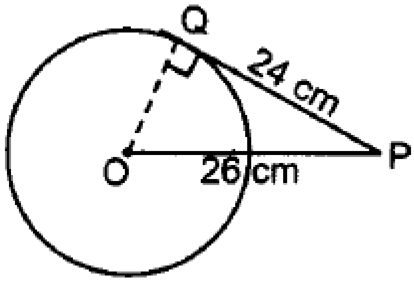
$\therefore$ P(getting an odd number)=$P\left(E_3\right) = \frac{3}{6} = \frac{1}{2}$.

iv. Let E_4 be the event of getting a number greater than 4.

Number of favourable outcomes = 2.

Then, the favourable outcomes are 5,6.

$\therefore$ P(getting a number greater than 4) = P(E_4)$= \frac{2}{6} = \frac{1}{3}$.

14. 

According to the question,$OP = 26\ cm\ and\ PQ = 24\ cm$

In $\triangle$OQP, we have $\angle$Q = 90°

$\mathrm{OP}^2 = \mathrm{OQ}^2 + \mathrm{PQ}^2$

$\Rightarrow (26)^2 = \mathrm{OQ}^2 + (24)^2$
$\Rightarrow \mathrm{OQ}^2 = 676 - 576 = 100$
$\Rightarrow \quad OQ = 10\mathrm{cm}$
$\therefore$ Radius of the circle$= 10cm$

Section D

15. i. Given:
Length of rectangle = 12 cm
Width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
Height of cylinder = 10 cm
Diameter of base = 7 cm
$\Rightarrow$ Radius of base = 3.5 cm
Volume of cylinder = $\pi r^2 h$
= $\frac{22}{7} \times 3.5^2 \times 10$ = 385 cm^3
ii. Given:
length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
Volume of wood scooped out = 2 $\times$ volume of hemisphere
$\Rightarrow$ Volume of wood scooped-out = $2 \times \frac{2}{3} \times \pi \times r^3$
$\Rightarrow$ Volume of wood scooped out = $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$
iii. length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
Total surface area of the article
$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$
$= 70\pi + 49\pi = 119\pi$
$= 119 \times \frac{22}{7} = 17 \times 22$
= 374 cm^2
OR
Given:
length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
T.S.A of cylinder = 2πr(r + h)
$\Rightarrow$ T.S.A of cylinder = $2 \times \frac{22}{7} \times 3.5$(3.5 + 10) = 99 cm^2

16. i. AR = x m
ii. Quad. ORBQ is a square.
iii. a. PC = 8 + x
$AC^2 = (8 + 2x)^2 = 49 + 225 = 274$
$\Rightarrow 8 + 2x = \sqrt{274}$

$\Rightarrow x = \frac{-8+\sqrt{274}}{2}$ or 4.28 approx.

OR

b. $AC^2 = (8 + 2x)^2 = 49 + 225 = 274$

$\Rightarrow 8 + 2x = \sqrt{274}$

$\Rightarrow x = \frac{-8+\sqrt{274}}{2}$ or 4.28 approx.

Hence, radius r = 7 - x = $7-\left(-4+\frac{\sqrt{274}}{2}\right)$

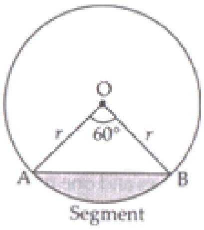
= $\left(11-\frac{\sqrt{274}}{2}\right)$ or 2.72 approx.

Therefore, radius of the circle is $\left(11-\frac{\sqrt{274}}{2}\right)$ m or 2.72 m approx.

Section E

17. Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



θ = 60°

OA = OB = r = 12 cm

$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow$ x + x + 60° = 180°

$\Rightarrow$ 2x = 180° – 60°

$\Rightarrow x = \frac{120^\circ}{2} = 60°$

$\therefore$ $\triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2\theta}{360^\circ} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14\times12\times12\times60^\circ}{360^\circ} - \frac{\sqrt{3}}{4}\times12\times12$

= $6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

18.

Age (in years)	No. of patients (f_i)	Mid point (x_i)	x_if_i
5 - 15	6	10	60
15 - 25	11	20	220
25 - 35	21	30	630
35 - 45	23	40	920
45 - 55	14	50	700
55 - 65	5	60	300
Total	**80**		**2830**

$\Rightarrow$ Mean $= \frac{2830}{80}$

= 35.375

Modal class = (35 - 45)

$\Rightarrow$ Mode $= 35+\left(\frac{23-21}{2\times23-21-14}\right)\times h$

= 36.81

Therefore, mode and mean of given data are 36.81 years and 35.375 years respectively.